# MEASUREMENT UNCERTAINTY EVALUATION IN THE DIGITAL ERA

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**Abstract:** Disadvantages of the traditional approach to measurement uncertainty evaluation are analyzed. An algorithm for measurement uncertainty evaluation based on the Monte Carlo method is described, which makes it possible to eliminate these shortcomings. The results of a comparison of measurement uncertainty estimates obtained by the traditional method and the Monte Carlo method are presented. The need to revise the “Guide to the Expression of Uncertainty in Measurement” is substantiated and its current state is described.
**Keywords:** measurement uncertainty, Monte Carlo method, revision of the “Guide to the Expressing of Uncertainty in Measurement”, kurtosis method

# ОЦЕНИВАНИЕ НЕОПРЕДЕЛЕННОСТИ ИЗМЕРЕНИЙ В ЦИФРОВУЮ ЭПОХУ

**Аннотация:** Анализируются недостатки традиционного подхода к оцениванию неопределённости измерений. Описан алгоритм оценивания неопределенности измерений на основе метода Монте-Карло, позволяющий устранить эти недостатки. Приводятся результаты сравнения оценок неопределенности измерений, полученные традиционным методом и методом Монте-Карло. Обосновывается необходимость ревизии «Руководства по выражению неопределенности измерений» и описывается ее современное состояние.
**Ключевые слова:** неопределенность измерений, метод Монте-Карло, ревизия «Руководства по выражению неопределенности измерений, метод эксцессов

## Introduction

International Metrology Day was held this year on May 20, 2022 under the motto “Metrology in the Digital Era”. This topic was chosen because digital technologies have revolutionized our community and are one of the most significant trends in the world today. And since the accuracy of measurements determines the reliability of scientific research, industrial production, and international trade, it is advisable to analyze the impact of digital technologies on approaches to assessing the accuracy of measurements.

In 1993, the “Guide to the Expression of Uncertainty in Measurements” (GUM) [1] were published, which were based on: the law of propagation of uncertainty, which leads, for nonlinear model equations, to a shift in estimates of the numerical values of the measurand and its uncertainty; the central limit theorem of probability theory with the apparatus of the number of degrees of freedom, predetermining the unreliability of the expanded uncertainty estimates due to ignoring the influence of the laws of distribution of input quantities on the law of distribution of the measurand.

To get rid of these shortcomings allowed the introduction of the so-called law of propagation of distributions, based on a numerical method – the Monte Carlo method (MCM) [2-4]. The MCM is based on the generation of input values in the form of random numbers with a given distribution law and their transformation through a measurement model into a set of random numbers with a distribution law corresponding to the distribution of the measurand.

The use of MCM was a real breakthrough in the measurement uncertainty evaluation, since it made it possible to get rid of the disadvantages of the traditional approach listed above. However, it turned out that the measurement uncertainty estimates obtained using the MCM numerically differ from the estimates obtained using the GUM approach.

Therefore, Supplement 1 to the GUM (GUM-S1) [4], setting out the MCM, came into conflict with the GUM itself. This placed the task of revising the GUM before the Joint Committee for Guides in Metrology (JCGM) Working Group WG1.

## The essence of the Monte Carlo method

In the Monte Carlo method, the input quantities $X_1, X_2, ..., X_N$ are represented as random variables with probability distribution functions (PDF) $g_1, g_2, ..., g_N$. The expected values and standard deviations of these PDF are equivalent to the estimates of the input quantities $x_1, x_2, ..., x_N$ and their standard uncertainties $u_1, u_2, ..., u_N$, respectively. In this case, the application of the Monte Carlo method consists in performing the following operations:

1) generating $N$ arrays of random numbers $x_j$, $j = 1, 2, ..., N$, large volume $M$ ( $M = 10^6$-$10^7$), obeying the given distribution laws;

2) obtaining an array of estimates of the output value $y$ of the volume $M$ ;

3) calculation of estimates of the parameters of the obtained distribution: mathematical expectation $y$, combined standard uncertainty $u(y)$, coverage factor $k$ and expanded uncertainty $U_p$.

## GUM revision

It should be noted that the following factors hinder the direct use of MCM for measurement uncertainty evaluation in testing and calibration laboratories accredited for compliance with the requirements of ISO/IEC 17025:2017:

• lack of specialized certified software tools for measurement uncertainty evaluation based on MCM;

• the impossibility of obtaining the measurement uncertainty budget by the existing software tools that implement the MCM;

• the impossibility of documenting a step-by-step procedure for measurement uncertainty evaluating based on MCM.

In addition, a comparison of the estimates of the combined standard uncertainty obtained using the approaches described in [1] and [4] shows their numerical difference, primarily due to the difference in finding the type *A* standard uncertainties of input quantities. This posed a challenge for WG1 GUM revisions [5].

The first draft NewGUM was circulated by the end of 2014 to JCGM Member Organizations, National Metrology Institutes and other recipients. The revised Guide retained the law of propagation of uncertainty, with Type *A* and Type *B* uncertainty estimates being calculated based on a Bayesian approach. However, the developers of NewGUM failed to offer an approach to a reliable expression of the expanded uncertainty.

## Kurtosis method

As part of the implementation of the topic COOMET 796UA19_2019, the Recommendation "Expression of the expanded measurement uncertainty (kurtosis method)" was developed, which makes it possible to obtain estimates of the numerical values and uncertainty of the measurement result that are close to the estimates obtained using the MCM.

The recommendations include the following sections:

1. Introduction.

2. Terms of use.

3. Notation used.

4. General provisions.

4.1. Measurement model.

4.2. Estimation of input quantities, their standard uncertainties and covariances.

4.3. Calculation of the numerical value of the measurement result.

4.4. Calculation of the standard uncertainty of the measured quantity.

5. Calculation of the expanded uncertainty.

5.1. Kurtosis method [6-7].

5.2. The law of propagation of expanded uncertainty [8].

Appendix A. Finding the parameters of distributions by coverage factor.

Appendix C. Accounting for the shift of the numerical value of the measured value in the case of a nonlinear model equation [9].

Appendix C. Accounting for the bias of the standard uncertainty of the measured quantity with a nonlinear model equation [10].

Appendix D. Student coefficients for the number of degrees of freedom ν and the probabilities 0.95 and 0.9545.

Appendix E. List of publications with examples of estimation of measurement uncertainty by the proposed methods.

Literature.

Draft COOMET Recommendations "Expression of expanded measurement uncertainty (kurtosis method)" was approved at the 33$^{rd}$ COOMET Committee meeting which will be held online on the Zoom platform on 25-27 October 2022.

## Rebranding GUM

In 2017, JCGM rebranded documents in its portfolio that were prepared or to be developed by Working Group 1: the entire set of documents is now known as the "Guide to the Expression of Uncertainty in Measurement" or "GUM", and deals with the evaluation and expression of measurement uncertainty , as well as its application in science, commerce, health, security and other types of public activities. The GUM will consist of the following 12 documents [11]:

- JCGM GUM-1. Part 1: Introduction.
- JCGM GUM-2. Part 2: Concepts.
- JCGM GUM-3. Part 3: GUM:1995 with minor corrections.
- JCGM GUM-4. Part 4: The role of measurement uncertainty in conformity assessment.
- JCGM GUM-5. Part 5: Examples.
- JCGM GUM-6. Part 6: Developing and using measurement models.
- JCGM GUM-7. Part 7: Propagation of distributions using a Monte Carlo method.
- JCGM GUM-8. Part 8: Extension to any number of output quantities.
- JCGM GUM-9. Part 9 Statistical models and data analysis for interlaboratory studies.
- JCGM GUM-10. Part 10 Applications of the least-squares method.
- JCGM GUM-11. Part 11 Bayesian methods.
- JCGM GUM-12. Basic methods for uncertainty propagation.

## Conclusions

The digital technologies that have revolutionized our community have not bypassed the theory of measurement uncertainty evaluation. An example of this is the development of a numerical implementation of the law of propagation of distributions based on the numerical Monte Carlo method. The use of MCM was a real breakthrough in the measurement uncertainty evaluation, since it made it possible to get rid of the disadvantages of the traditional approach listed above. However, it turned out that the estimates of measurement uncertainty obtained using the MCM are in poor agreement with the estimates obtained using the GUM approach. In this regard, there is a need to revise the GUM based on the Bayesian approach. Working group number 1 on manuals in metrology has not yet fully coped with this task in terms of estimating the expanded uncertainty. The way out of this situation is the method of kurtosis, brought to practical use by the author of the report. The recommendation developed on the basis of this method can adequately supplement the set of GUM documents proposed by WG-1 as part of its rebranding.

## Literature

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[10] **Zakharov I., Neyezhmakov P., Botsiura O.** Reduction of the bias of measurement uncertainty estimates with significant non-linearity of a model equation // Journal of Physics: Conf. Series 1379 (2019) 012013.

[11] **JCGM GUM-1** Guide to the expression of uncertainty in measurement – Part 1: Introduction.

Метаданные статьи:

1.Полное название статьи: Measurement uncertainty evaluation in the digital era (Оценивание неопределенности измерений в цифровую эпоху);

2.Укороченный вариант названия статьи (Runningtitle): Measurement uncertainty (Неопределенность измерений)

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